



EARLY MATH CONCEPTS

@raisingreaders



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Dear Parent/Caregiver:

Throughout my years of doing this work, I have worked with a range of parents, caregivers, and educators from all walks of life. One thing that I have always found interesting is how little attention developing **early math skills**, particularly through play, is given to Black and Brown children. It is as if, concerned with literacy and the literacy crisis in our community, we forget math and numeracy altogether.

For most parents, the approach to math the early years is simple: teach them to count and the basic shapes and hope for the best. On the other end of the spectrum, some parents wanting to give their children a competitive edge, start drilling facts and worksheets into toddlers and preschoolers not realizing that for our youngest citizens, this approach counters all of the science we have on how children learn and the develop through the early years. **In the end, we end up with missed opportunities.** This is then reflected in reduced interest and ability in the areas of math and science; which consequently, also results lost wealth *and joy* for individuals and the collective.

In other words, in the same way, that low literacy affects our community's ability to thrive, so does low numeracy, both with life long implications as careers and vocations with a greater likelihood of higher pay and wealth accumulation are usually mathematically demanding.

I mention joy because too often we forget that in the same way that our children are making connections about concrete subjects like phonics and one-to-one correspondence, young children are also making connections about the *learning process* and what it is it. Through their environment, they are absorbing what it means to *learn*, its importance, and how to do it. They are learning about personal accomplishment, joy, fulfillment, and the value which these have. It is all learned early on.

By adequately preparing children to develop a deepened love and intrinsic motivation, we prepare them for a life of agency. Our children should BOTH be able to do advanced work throughout their lives and enjoy or at least want to do it. They should have positive relationship with learning so that they are able to pursue whatever career and life paths they are purposed for. In giving children the space and support to develop comprehensive literacy and numeracy skills, through play and mastery, we empower them with BOTH ability AND desire. If we do not prepare them, learning will be difficult and they will be unable to do advanced work. But, if our preparation feels oppressive, though they might be able, they will be unwilling. Inability and lacked desire are equally damaging and should both be avoided.

When we think about the things we want for our children, as Black and Brown parents, we often think in terms of financial stability alone. This is understandable, given our history in this country. We want and deserve greater wealth. But, in that process, we forget that joy, fulfillment, and purpose are also defining factors of success. Perhaps, because we have been denied these things by the generations before who have themselves, too, been denied. This experience is a byproduct of systemic racism, of oppression, of colonization. To this effect, I share with you, that without envisioning and and embracing a more comprehensive idea of success for our children, which includes financial success but also deep emotional grounding and joy, we, as parents super impose an oppressive state on our children's lives. What a tragic realization -- that we, as caregivers, can unknowingly be both, oppressed and oppressor... all while loving our children!

I share these things with you because I love you. I deeply care about us as parents and caregivers. I believe for us the call is greater than white, privileged counterparts raising white children. They just have to teach their children. We have to liberate them, even when we are also learning to get free.

The good news is that there is a way. We can do both. By following best practices early, through play, inquiry, and intentionality, we can develop a love for learning and healthy mastery of concepts.

This is the vision. Thank you for allowing me to walk with you.

In Service,

Nikolai

There is math everywhere.

You're children are already using and exploring mathematical concepts every day and will further this through play and concrete experiences. Void of either or both of these, math for children is another pointless external task to complete for the sake of some construed hierarchy of acceptance, ie, because mom/dad/teacher requires it. This might or might not work out for your child. But seldom does lead to deep mathematical reasoning. In order to support the development, mastery, confidence, and appreciation for math, it must be anchored within the child's wiring. A good math teacher can make a great difference on any one given grade or topic but the constant is the home environment and therefore it is in the home environment where "math sense" and "mindset" is best developed.

This short guide is here to accompany the classes which you have taken with me.
Refer to it often until you have internalized it. Find the math, see it, reinforce it, cultivate it.

Math Tools in Your Home

Math is all about NUMBERS. It is about quantifying and qualifying data. We count numbers. We track them. We get information from numbers. We then use that information. There is utility in them. Numbers enumerate, quantify, and organize. Right?

As adults, we know these things. But, children do not. They do not because we do not teach them the utility of the math. We just expect them to compute it at some point. Again, without an understand of utility, children may or may not be able to do it with ease or be motivated to want to do.

What can parents do? For starters, just like in the literacy program, where I stressed the need to create a literacy centric experience for children in the home, we must also create a numeracy centric experience in our homes as well.

Here are tools to have in your home and USE explicitly:

- Measuring tape and scale
- A growth chart (height/weight). Weight and measure your child. It has utility and it is nostalgic. It is an activity to contextualize. However, please very mindful of judgement for height and weight. When tracking height and weight demonstrate the act of tracking data. That's the sole purpose.
- A calendar (at eye level), for young children. Having a calendar that keeps track of the seasons is a wonderful tool as well but, I like using a traditional calendar as well. Calendars that let us flip back and forth teach us about the continuity of time, the relationship of numbers in a "number line", record keeping, etc. We can also write on them and cross numbers out. It is different than the "universal" calendar with the seasons.
- Kitchen tools: food scale, measuring cup, and measuring spoons. Pots and pans are also math tools as different pans with different surface areas.
- A thermometer
 - outdoor (These are fun for making observations and aid agency. When does the temperature go up/down/stay the same? what does it affect?)
 - indoor (usually your thermostat has this but children can't see it/reach it)
 - personal temperature (one that you feel comfortable with children using regularly in addition to the one for personal emergencies)
- Clock (digital and analog for younger children) and timers (kitchen and sand). Clocks and timers should be at eye level and accessible.
- Rain water gauge: measure informally to begin with but also chart and compare and contrast as time goes by
- Stop watch
- Sundial (optional and of course diy)
- Obvious: ruler, calculator, protractor, etc.
- Maps
- Different number frames (example: a carton of eggs can be turned into a 5 or 10 frame and can also be used as a 12 or 6 array)
- Abacus
- 3 dimensional figures
- Less obvious/most underused tools: Sticks, rocks, leaves....words, songs, and books but most of all...observations!

If you look around, you probably have all or many of these at home already. Now, use them with intentionality.

Use math language

Perimeter, area, surface, face, sphere, cone, rectangle, infinite, long, short, above, below, through, circular, pattern, opposite, curved, line, fifth, bigger, shorter, deeper, value, order, estimate, structure, speed, fast, aperture, percent, discount, special, sale, slope, clockwise, counter clockwise, in addition to, half full...there are so many math reasoning words for us to use. *Use these words in context, start conversations about them, weave them into joint play.*

Math books and songs:

- Book Series: MathStart
- Book: How Much is a Million?
- Book: Feast for 10
- Multiplication Rocks (itunes and youtube)
- Multiplication and booklets from the company “The Good and the Beautiful”
- Alice the Camel
- 5 Little Ducks
- 5 Little Monkeys (jumping on bed or sitting in tree)
- 5 Speckled Frogs

Math Curriculum Materials

- Base ten blocks (Math U See integer set is a good one)
- Number Neighbors (by company Muggins Math - as you get older, extension to pre-algebra is available)
- SumBlox (*these take up space and the least versatile but are pure joy*)

Introduce math reasoning and problem solving

“I only have one left and there are two of us, what can we do?” “I have 3 things and one backpack, can you help me fit them in my bag?” "Can we move these things around and to help us fit them in?"

Don't miss opportunities to make math *real*.

Writing Numbers:

Just like letters, your child will be ready to write numbers, once his/her hands are ready and they are MOTIVATED to write them because you have given them utility through modeling and concrete experiences.

Prepare your children's hands for writing through play and give them a REASON to WANT to write and use numbers. Model number writing by using numbers and writing them in front of them. Label, enumerate, write the numbers as you say them, record things with numbers. The more excited you are about math and the more utility you give it, the more eager your preschooler will be to write numbers. Again, like we do with letters, have your children interact with 3 dimensional numbers and create or build those numbers by tracing in sand or forming them with beans, pebbles, pasta, etc. This will aide them in transitioning to number writing.

Above all: model utility. Make charts, make tally marks, write numbers with chalk, write numbers in sand, write in a calendar, write page number, write dates, write checks...model, model, model.

Math and Play

When you are playing with children, insert math reasoning and games in the play. When you are preparing the environment, prepare it such that it lends itself to math reasoning. But if you are observing independent play, use judgement. It is not always best to insert ourselves. We can always take pictures, or "document" moments and then, talk to children about those experiences in ways that incorporate math reasoning.

In your day-today interactions, incorporate math into your routines and games by scoring, timing, recording, charting, “paying” for things, “borrowing” / “lending” things, measuring, following directions/recipe, drawing a map/following a map, etc. You can also go on walks or explorations and bring basket or bag to collect things. Count the things you have collected, compare them, measure them, feel them, discuss their shape, sort them, create patterns with them, split them/bend them to make fractions, build with them or build a home for them, etc. These are all opportunities to make concrete connections with numeracy.

Parts and Whole

From fractions to addition and subtraction to prime factorization, a key math concept is that numbers are made up of other numbers. Bringing attention to parts of whole things is the beginning of this understanding. A body has parts. So does a flower and a lego sculpture. Two halves of a sandwich make up one whole. A puzzle has pieces which will make one picture. Become aware of how important this concept is and all the ways which you can expose your child to parts/whole through play and incidental interactions. It is the groundwork for advanced mathematical reasoning.

Patterns

Understanding patterns is a part of mathematical reasoning (science, human behavior, and literacy too). Estimation, predictions, and problem solving are all areas in which understanding patterns helps us. Activities where we can find and build patterns are useful but we can also start with just noticing or actively looking and finding patterns. One quick example: finding patterns in our clothes/fabrics or in nature. Another fun one is noticing patterns in songs. There are many others. By noticing and finding patterns, we give credence to their importance.

Using patterns to solve problems is another way to make them important. Why would a child want to work on activities of something that have zero relevance? Concrete experiences help children make the connections and those connections turn into motivation.

One to One Correspondence

Work to make sure children understand that the number 3 represents 3 things, $5 = 5$ objects, unless you add or take objects so that the number of objects changes, the number always stays the same even if you move the objects around.

Counting/Order

As children learn to count, have them make the connection that the next number is one number one, than previous one. Then, explore a more comprehensive relationship with counting. Count forward and then backwards, count to a number but not starting from 1. For example, count to 18 starting from 6. Ask questions like "what number is 2 more than 7?" and play with getting to that answer. Having a deeper understanding of numbers and counting is more important than just counting indefinitely. Take your time and build this knowledge.

Counting, counting, counting...so many ways to practice!

- Sing (skip count)
- Hopscotch
- Hide and seek (forward and backward)
- Count up and down in a line, or horizontally
- Count scattered things or in a pattern or in a line (always have fun)
- Count as you are jumping rope
- Play a game of counting forward then switching and counting backwards with a code word for the switch
- Count page numbers or add page numbers to your homemade books
- Put 3 dimensional numbers in order, paint numbers on rocks
- Count hops, count skips, count as you climb up steps...
- When you are out doing errands or going on a walk, "collect" and "record" numbers in a number hunt
- counting forwards and backwards is the base of addition and subtraction
- skip counting is base for multiplication
- counting bundles of ten gets us to very large numbers very quickly and is FUN!
- tally marks are the beginning of counting/multiplying by 5s and also help us count up very quickly
- Find things in nature that have a specific number of things and use them as a model to skip count. Birds have two feet (and people!), some mamas have 4 feet, spiders have 8 legs, and so forth.
- Use 10 frames, egg cartons, or other arrays to split any number of small objects evenly, this is the base of division.

Favorite Counting Game

This very simple game builds algebraic thinking and will help with beginner word problems.

Once children have an understanding of even just a few numbers, start playing "what's under the cup." This game can be played with large numbers but also with numbers as little as 1-4. Have one number out, say the number 4, show your child what the number is. Without the child seeing, break up the number into two quantities and hide them under two cups. Reveal one of the cup's quantity, say 3. Now ask: What's under the cup? The answer/guess should be 1. The child can then check by raising the cup. Have fun!

Function of numbers

Numbers tell us things. Addresses, coordinates, page numbers, days of the calendar, measurements, serving size, measuring tape...get into it. What are these numbers telling us? Share it. Chart it. Make it real.

Subitising

Di/dice faces, dominos, pairs, 5 frame, 10 frame, tally marks, hand positions...etc.

Play games that allow children to memorize what numbers in a pattern look like quickly. They will use this skill to then visualize groups of things, quantify, and estimate quantities quickly.

Time

Developing an interest in time keeping and having a sense of what time *feels* like is a much more valuable skill and merely reading a clock. An understanding in how long 5 minutes, 1 hour, or 1 week feels like is grounding to young children. Concrete experiences that demonstrate what it feels like to wait, measure, and estimate time and why we even do it are to be prioritized.

Before they can read a clock, let children feel time with their bodies and experiences. It is important to be given space to observe how the sun moves throughout the day; to see how shadows are impacted by time of day and the location of the sun. It is a beautiful thing for children to understand the days of the week and months in the season because they have actively used a calendar and counted down the days for a special event.

When visiting places where time matters like a bus station or airport make sure you are intentional in how you frame time. Take walks of the same places at different times of the day and seasons and notice difference. Use the words clockwise and counter clockwise. Keep track and look forward to fruits and vegetables that are in season. There are so many ways to make the time real.

Playing, observing, tracking, and documenting, all give ground to children's relationships with time. If children can count (by one and 5s), and they know what the number's names are, they can be taught to read an analog and digital clock. *It will come with time (no pun intended).*

Estimation

Estimating is an important skill in mathematics and day to day life. We estimate when we cook, budget, and bid for contracts. It's everywhere.

People who are good at subitising tend to be good at estimating. But subitising is not the end all. Experiences are another important component. Estimating is truly one of those things where the more you experience, the better you are at estimating. Virtually every experienced contractor, regardless of the field, will say, early on in their career they ran into over or under bidding. Experience made the difference .

In the same manner, the more opportunities children get to play and the richer their experiences, the better they will be at estimating. For example, the more structures they build, the easier they can estimate how much material they will need to build a new structure. If a child measures a cup and then gets to feel what that cup feels like as a handful, they can estimate quantities in "handfuls" later.

Estimating as a skill takes time and repeated action to learn. Make estimating something fun and relaxing. "Estimate" how long/how much things will be. Write down or take turns estimating/guessing. See who was closer. Talk about strategies for estimating, which strategies were helpful, which were not, etc. The more they do it, the easier and more accurate they will get. Because estimating is not an exact science however, children will also need to feel comfortable with being wrong/taking risks. Being able to take risks is developed through play and independence, a nurturing relationship between caregiver and child, and the practice of correcting children without judgment. Children that feel they are trusted able, feel safe to take risks.

Greater than/Less than/Equal

Long before children know anything about the numerals (1, 2, 3...5, etc.), as young as 6 months, studies show they can tell quantities that are greater and less than. Build on that.

Children work on this skill through independent play all the time. While lifting things will figure out what things weigh more or less, what containers holds more liquid, what plate has more or less snacks.

When you are setting up the environment, you can make strips of ribbons or tape of different lengths and widths and compare which is greater or less than. As children get older add numerals and less variation of size or quantity for greater challenge. Keeping a little roll of Washi tape in your purse to play is great for dining out and keeping little one's busy. Practice pouring water (add food coloring for fun) and compare which cups have greater, equal, or less liquid. Build number stairs out of toys or natural objects. These experiences will allow children to see and feel what is greater than, lesser than, or equal. This is the foundation of estimating, adding, subtracting, inequalities, etc.

If you are working with base ten blocks or any other three dimensional materials, you can also make a visual connection for greater, lesser, and equal. Even in your direct instruction, it is important to account for movement, enjoyment, and correction without judgment.

Geometry

From design to problem solving, geometry is *everywhere*! Yet, this is one area where many of us limit children's exposure. Too often, our approach to young children and geometry is quite simply to teach them basic shapes. In the process, we miss many opportunities for internalized deep understanding.

For starters, we interact with three dimensional shapes like spheres, cones, or cubes every day but seldom do we teach children their names and properties. Next, let's think about spatial reasoning and problem solving. This too is geometry. What shapes work best for a particular objective or purpose? Why are most lenses in magnifying glasses or binoculars round? Where can we find shapes and angles in nature and in real life? How much space do we need to get through a tight space? How does that vary? Does mom need more space than your friend or less? How much paper do we need to wrap a present? How many toys will fit in a backpack? Does the order in which we place items in a tight space make a difference? It is insane how often geometry shows up in our daily lives. Isn't it a disservice to not teach children language and context for something so widely available and useful?

There are many examples where we can use geometry and spatial reasoning language and problem solving through direct instruction, environment provocations, or "after the fact" play reflections.

Fractions

Drawing attention to part and wholes introduces the idea that "things" are units which are made up of other units. This is loosely related to fractions. But fractions are not just parts of the whole, they are equal parts, which calls us to be specific. Before we get too into this section, I want to make sure I state this: we are not expecting very young children to do anything that resembles what we visualize children "doing" or "understanding" when we think: **fractions**. Don't break out the flashcards or get too crazy about this.

What I want you to understand is that many children struggle with fractions later because they are missing underlying connections from their early experiences and/or hold limited beliefs about themselves and their mathematical ability. With this in mind, I want you to be encouraged. Because with the tools you are gaining now, you are in fact, able to support play based development that will result in your child's abilities later on. Let me illustrate this with a little more clarity.

One of the connections about fractions that children (and maybe even adults) struggle with is that a fraction is an expressed division problem. But what exactly is $\frac{1}{3}$? It is one thing split into or divided by three parts. Right? It's pretty straight forward. So, if we go on a walk with our children and gather 9 pinecones and then come home and divide them into 3 groups, we would learn that there are 3 pinecones in each group, or *third*. We could combine two of those groups and observe that those groups now add up to 6 pinecones, *two thirds*. And, if we put them all together and make one big group again, we end up with the 9 pinecones. *The three thirds are the same as the original one whole*. This is just one scenario but there can be many more. Even without a single symbol or numeral, this is pretty impressive for preschoolers.

Because the children can skip count, because they have put things together into arrays like 10 frames, because they have grouped evenly through one to one correspondence, because can subitize, because the language has been used over time, the little ones begin to follow the math in an organic way. I have seen and worked with children as young as 5 that can do what schools deem are 2nd and 3rd grade word problems and they are doing it all through play and nurturing interactions.

Cut things into halves, quarters, and thirds. Combine fractions into wholes. Pizza slices and mandarin oranges are great math materials! Use the language. If you want to introduce a more "formal" material, use Montessori skittles or plastic circles which are cut into difference fraction pieces. Label and chart things using fractions once children are comfortable with whole numbers. Simple fractions are part of every day life. Children are part of every day. The two can coexist beautifully.

Money

People often ask me how to teach young children about money and when to start. My approach is pretty straight forward. First, you model it's use. Less cards, phone scanning, and online option is the first step. A simple three-period lesson with actual money or images is next. Then, lots of play activities, a piggy bank, using the borrowing/lending/payback framework with/without money, letting people pay at the store, skip counting using nickles and dimes, then \$5, \$10, and \$20s, the usual drill. Modeling, utility, play, concrete actions, and joy.

Miscellaneous Math Concepts and Games

There are lots of other concepts which you can continue to build on. Here are some additional ones:

- Explore lines (vertical, curved, zig zag, horizontal) and related language like intersections, vertices, and points
- Examine and evaluate which trees you can hang on and climb, which you cannot, how high you can go, and why
- Fill up balloons with water and different objects (cups, syringes, facet, etc). How much water do different balloons hold? How are balloons filled up faster?
- Create nooks and crannies for children to crawl, gather, and hide special things (spatial reasoning)
- Dig holes to represent below ground level (negative number)
- Fill holes with toys/objects and challenge children to estimate how many items go into it, how fast they can empty by counting by twos or threes or with larger/smaller containers or heavy objects versus lighter objects. Fill them with water and see which items make a bigger splash and empty out the water faster or how different size containers can affect how fast they can fill and empty it out
- Search nature and the internet for symmetry and/or geometrical shapes
- Create a zen garden (large or miniature) to explore lines and curves
- Play tic tac toe
- Building a sand castle (with just the right amount of water to compact the sand!)
- Play board games like chutes and ladders
- Race, keep log of time, estimate how far a ball would roll, explore how high a ball would bounce...talk about conditions/shapes which affect it rolling and bouncing, make predictions
- Play games of probability such as rock paper-scissor-shoot and head or tails (chart and keep score)
- At a pumpkin patch or any outdoor space, find big/little things, heavy/light things, things of same color/different color, match, create patterns
- Pouring water or liquids into different containers
- Build a DIY balance OR just find a traditional see-saw and add some intentionality
- Match socks, sort silverware ... from integers to primes, sorting and categorizing will be a part of mathematics, develop the skill with every day tasks and have fun!

As you can see, the math is there. Too often we do not develop our children's mathematical minds because we are not exploring through play and creating profundity in the reasoning. We are also not using and valuing math with our language and our intentionality. If we create early childhood environments that present problem solving and math concepts as values and of utility and we go beyond counting and recognizing numbers, our children will internalize concrete math experiences and develop the "math sense" that evades so many American children.

Strong literacy and numeracy skills through concrete, play based experiences and problem solving will go further than apps, worksheets, and flashcards every time.

For more ideas stay tuned to our FB group. I have also created this **[pinterest board](#)**.